

Methods and Tools for Analysis of Symmetric Cryptographic Primitives

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14th of October, 2014

Main goal

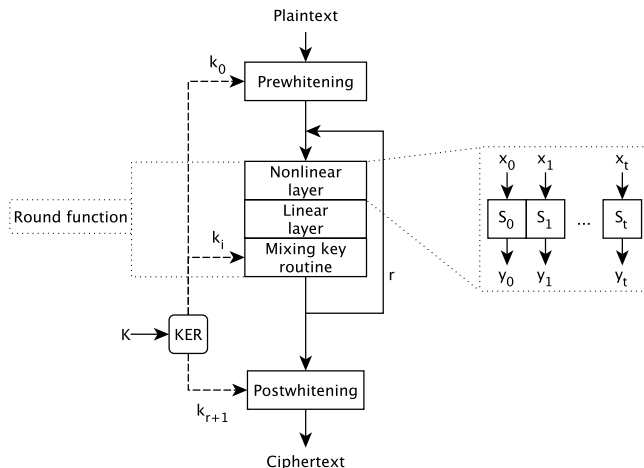
Improving the resistance of modern iterative cryptographic primitives to advanced attacks through the development of methods and tools of cryptanalysis.

National and international competitions

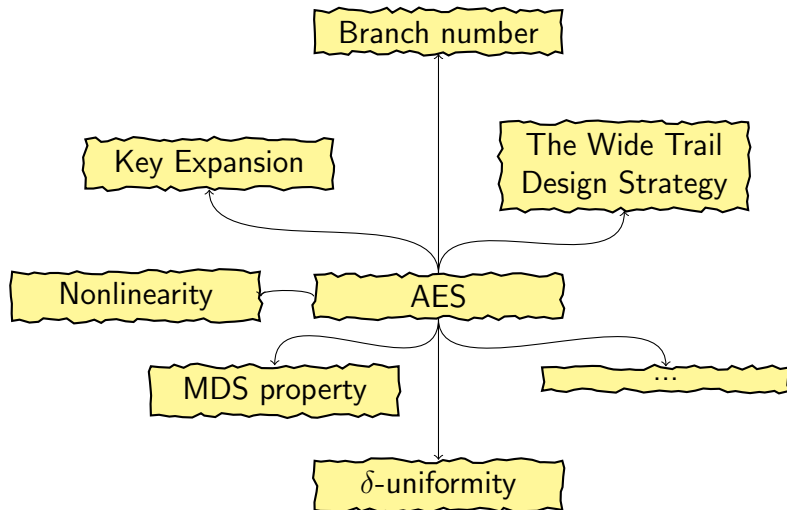
- Advanced Encryption Standard (1997-2001)
- New European Schemes for Signatures, Integrity and Encryption (2000-2003)
- eSTREAM (2004-2008)
- CRYPTREC (2000-2003-...)
- Ukrainian open competition to design a prototype of a block cipher for the new standard (2006-2009)
- SHA-3 (2007-2012)
- Closed competition to develop an advanced hash function and block cipher (2010-2012, 2013-...)
- Competition for Authenticated Encryption: Security, Applicability, and Robustness (2014-...)

An iterated block cipher

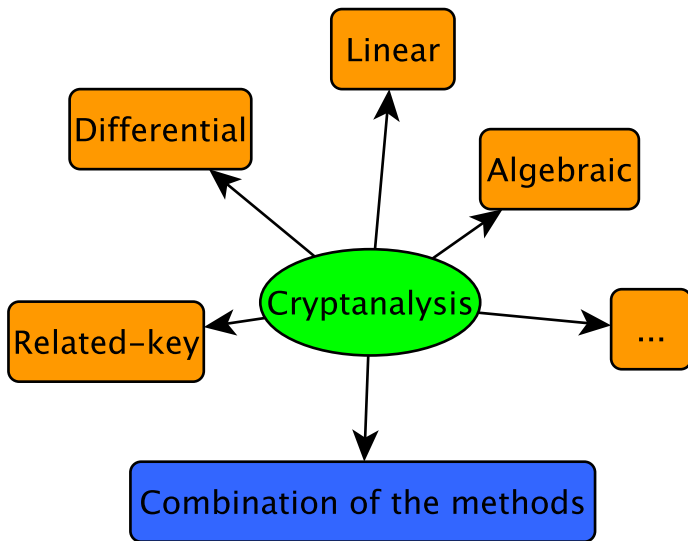
A **block cipher** encrypts a block of plaintext or message M into a block of ciphertext C using a secret key K .



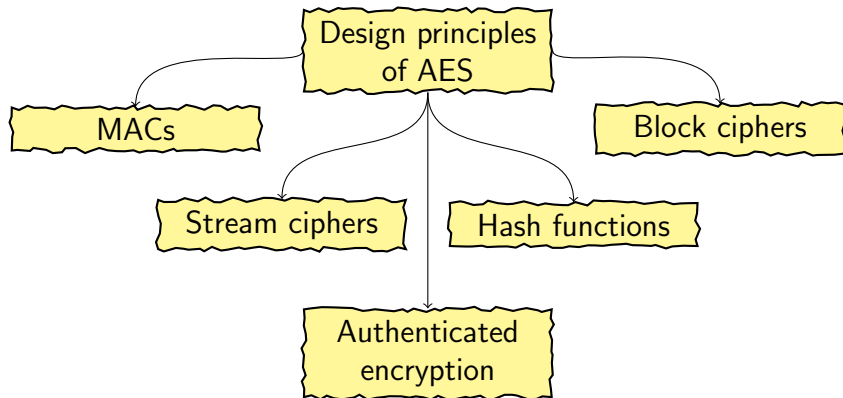
New design principles



Methods of cryptanalysis



Next generation of cryptoprimitives



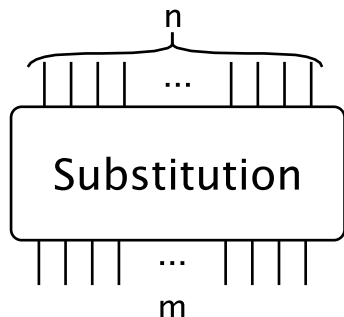


Figure : A Substitution Box

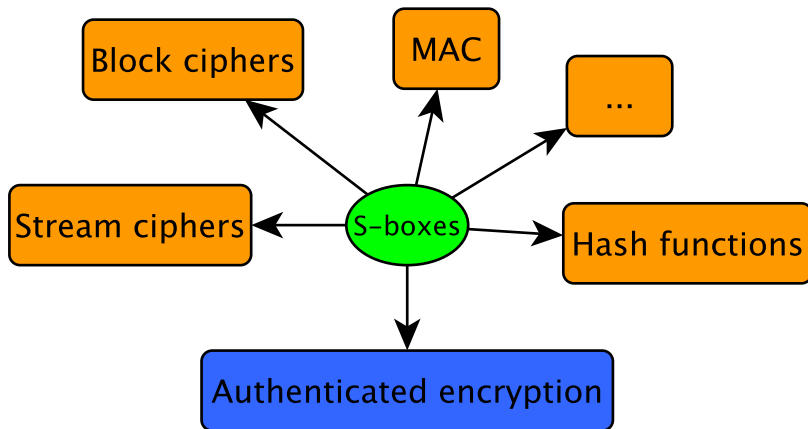
Possible variants

- $n > m$
- $n < m$
- $n = m$
 - $\#img(S\text{-box}) = 2^n$

Representations

- lookup tables
- vectorial Boolean functions
 - A set of Boolean functions
- system of equations

Application of substitutions



Cryptographic properties of S-boxes

Definition

An S -box is a mapping of an n -bit input message to an m -bit output message.

- Minimum degree
- Balancedness
- Nonlinearity
- Correlation immunity
- δ -uniformity
- Cyclic structure
- Algebraic immunity
- Absolute indicator
- Absence of fixed points
- Propagation criterion
- Sum-of-squares indicator
- ...

- Two functions F and G are called **EA-equivalent** if

$$F(x) = A_1 \circ G \circ A_2(x) + L_3(x)$$

for some affine permutations $A_1(x) = L_1(x) + c_1$,
 $A_2(x) = L_2(x) + c_2$ and a linear function $L_3(x)$.

- Functions F and G are **restricted EA-equivalent** if some functions of $\{L_1, L_2, L_3, c_1, c_2\}$ are in $\{0, x\}$
 - **linear equivalent**: $\{L_3, c_1, c_2\} = \{0, 0, 0\}$
 - **affine equivalent**: $L_3 = 0$

EA-equivalence

For $F, G : \mathbb{F}_2^n \mapsto \mathbb{F}_2^m$ another form of representation of EA-equivalence is the matrix form

$$F(x) = M_1 \cdot G(M_2 \cdot x \oplus V_2) \oplus M_3 \cdot x \oplus V_1$$

where elements of $\{M_1, M_2, M_3, V_1, V_2\}$ have dimensions $\{m \times m, n \times n, m \times n, m, n\}$.

Matrices M_i and vectors V_j have a form

$$M = \begin{pmatrix} k_{0,0} & \cdots & k_{0,n-1} \\ k_{1,0} & \cdots & k_{1,n-1} \\ \vdots & \ddots & \vdots \\ k_{m-1,0} & \cdots & k_{m-1,n-1} \end{pmatrix}, \quad V = \begin{pmatrix} v_0 \\ v_1 \\ \cdots \\ v_{m-1} \end{pmatrix}.$$

Algebraic Attacks Using Binary Decision Diagrams

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BalkanCryptSec'14
October 16, 2014

Binary Decisions Diagrams (BDDs)

$$f(x_1, x_2, x_3) = x_1x_3 + x_1 + x_2 + x_3 + 1$$

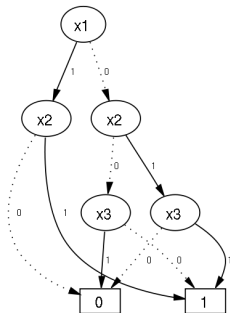
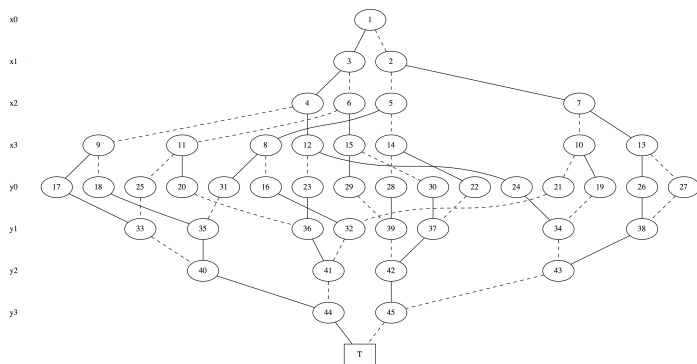


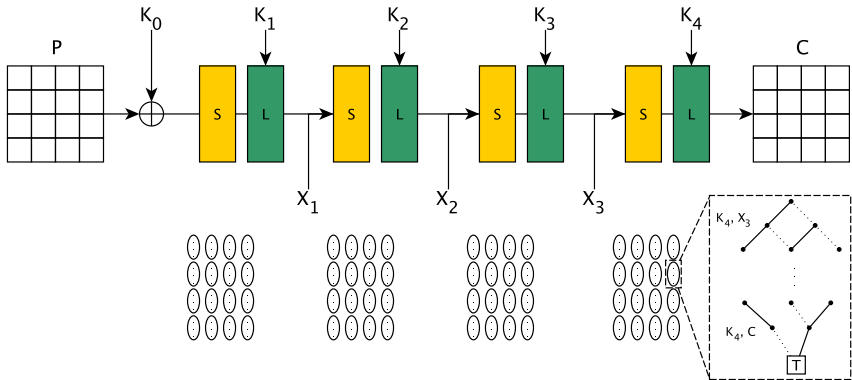
Figure : Binary decision diagram for the f function

S-box representation using BDD

S-box = {5, C, 8, F, 9, 7, 2, B, 6, A, 0, D, E, 4, 3, 1}



Description of 4-round AES



Digital Encryption Standard (DES)

- 2007: a system of equations for 6-round DES solved with **MiniSat** in 68 seconds (Courtois & Bard)
 - But ... necessary to fix 20 bits of the key to correct values
- **BDD method** allows to solve the 6-round DES in the same time without guessing (8 chosen plaintexts)

# texts \ rounds	1	2	3	4	5	6	7	8
4	$2^{22.715}$	$2^{14.506}$	$2^{10.606}$	$2^{10.257}$	$2^{9.805}$	$2^{10.070}$	$2^{10.203}$	$2^{10.381}$
5		$2^{22.110}$	$2^{16.455}$	$2^{13.526}$	$2^{13.995}$	$2^{14.212}$	$2^{14.410}$	$2^{14.704}$
6						$2^{24.929}$	$2^{22.779}$	$2^{20.571}$

Table : Complexities of breaking reduced DES

MiniAES

- There is no previous algebraic attacks for 10-round version
- The best know attack is only for **2 rounds**
- BDD approach allows to break **full version** of MiniAES using only 1 chosen plaintext

Rounds	4	5	6	7	8	9	10
Complexity	$2^{22.404}$	$2^{23.051}$	$2^{23.440}$	$2^{24.154}$	$2^{24.217}$	$2^{24.862}$	$2^{24.961}$

Table : Complexities of breaking MiniAES

Finding EA-equivalence

#	n	Number of solutions	Seconds used to solve		
			BDD	GB	SAT
1	4	2	$2^{4.05}$	$2^{1.30}$	$2^{13.71}$
2	4	60	$2^{4.86}$	-	$2^{16.77}$
3	4	2	$2^{3.92}$	$2^{1.01}$	$2^{12.08}$
4	5	1	$2^{10.20}$	$2^{11.43}$	$> 2^{18} \dagger$
5	5	155	$2^{10.48}$	-	$> 2^{18} \dagger$

\dagger not finished after 78 hours

Summary

- New approaches to algebraic attacks development
- The BDD approach allows to reduce complexity of algebraic attack on DES by 2^{20}
- Firstly was presented practical algebraic attack on 10-round MiniAES
- In some cases the BDD method is more universal and shows the best results compared to known methods

A Sage Library for Analysis of Nonlinear Binary Mappings

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CECC'14
May 21, 2014

Definition

An S -box is a mapping of an n -bit input message to an m -bit output message.

- Minimum degree
- Balancedness
- Nonlinearity
- Correlation immunity
- δ -uniformity
- Cyclic structure
- Algebraic immunity
- Absolute indicator
- Absence of fixed points
- Propagation criterion
- Sum-of-squares indicator
- ...

Design principles

- Orientation on arbitrary n and m
- Code optimization for performance
- Implementation of widely used cryptographic indicators

Generation of substitutions

- Gold
- Kasami
- Welch
- Niho
- Inverse
- Dobbertin
- Dicson
- APN for $n = 6$
- Optimal permutation polynomials for $n = 4$
- Polynomial
- ...

Unification of the functions

`generate_sbox` calls different methods based on parameters `method` and `T` which define generation method and equivalence respectively.

Additional functionality

- Extra functions
 - Resilience (balancedness and correlation immunity)
 - Maximum value of linear approximation table
 - APN property check (optimized)
- Convert linear functions to matrices and vice versa
- Apply EA- and CCZ-equivalence
- Generation of substitutions
 - Based on user-defined polynomial (trace supported)
 - Random substitution/permutation
 - With predefined properties
- Input/output
 - Set and get S-boxes as lookup tables
 - Get univariate representation/system of equations
 - Convert polynomial to/from internal representation

Performance

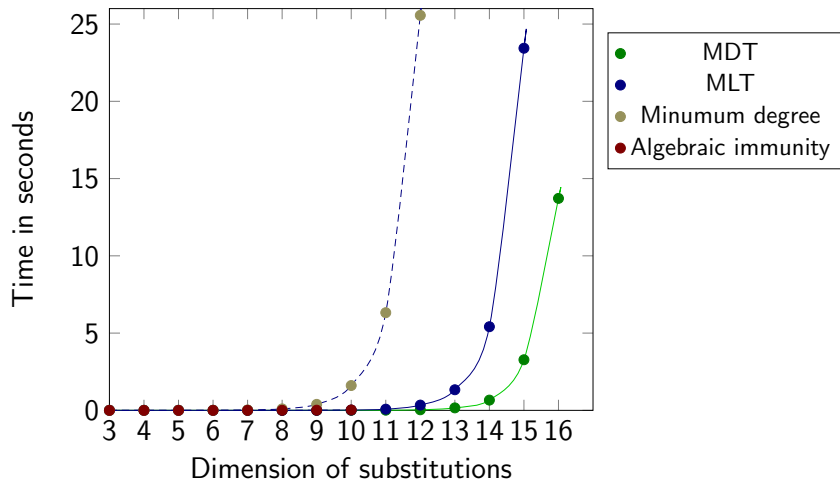


Figure : The relationship between dimension of random substitutions and time of calculation

Summary

- A high performance library to analyze and generate arbitrary binary nonlinear mappings
- Lots of cryptographic indicators and generation functions are included
- Functionality can be expanded quite easily
- Under development
- Hard to run for the first time
 - Works only in consoles
- Source code: <https://github.com/okazymyrov/sbox>

A Method For Generation Of High-Nonlinear S-Boxes Based On Gradient Descent

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CTCrypt'13
June 24, 2013

Definition

Substitutions satisfying mandatory criteria essential for a particular cryptographic algorithm are called optimal.

An optimal permutation for a block cipher has

- maximum value of minimum degree
- maximum algebraic immunity
 - minimum δ -uniformity
 - maximum nonlinearity
 - without fixed points (cycles of length 1)

Example of criteria

An optimal permutation without fixed points for $n = m = 8$ must have

- minimum degree 7
- algebraic immunity 3 (441 equations)
- $\delta \leq 8$
- $NL \geq 104$

Proposed method

Definition

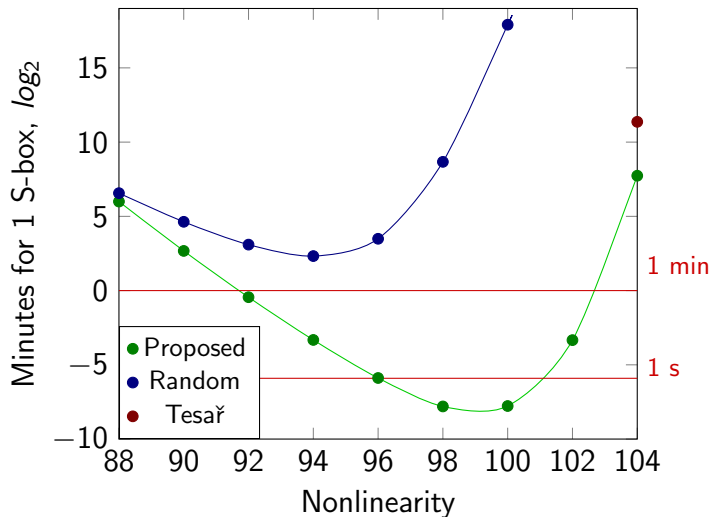
F is a highly nonlinear vectorial Boolean function with low δ -uniformity.

Example: $F = x^{-1}$ and $NP = 26$ for $n = m = 8$.

Algorithm

- 1 Generate a substitution S based on F .
- 2 Swap NP values of S randomly and set it to S_t .
- 3 Test S_t for all criteria depending on with the least complexity. If the S-box satisfies all of them except the cyclic properties then go to 4. Otherwise repeat step 2.
- 4 Return S_t .

Performance of practical methods



Comparison with known substitutions

Properties	AES	GOST R 34.11-2012	STB 34.101.31-2011	Kalyna S0	Proposed S-box
δ -uniformity	4	8	8	8	8
Nonlinearity	112	100	102	96	104
Absolute Indicator	32	96	80	88	80
SSI	133120	258688	232960	244480	194944
Minimum Degree	7	7	6	7	7
Algebraic Immunity	2(39)	3(441)	3(441)	3(441)	3(441)

Table : Substitutions comparison

Summary

- The analysis shows that both theoretical and random methods fail in case of optimal substitutions.
- The proposed method has the highest performance among the known methods available in public literature.
- Application of the proposed method allows to generate optimal permutations for perspective symmetric cryptoprimitives providing a high level of resistance to differential, linear and algebraic cryptanalysis.

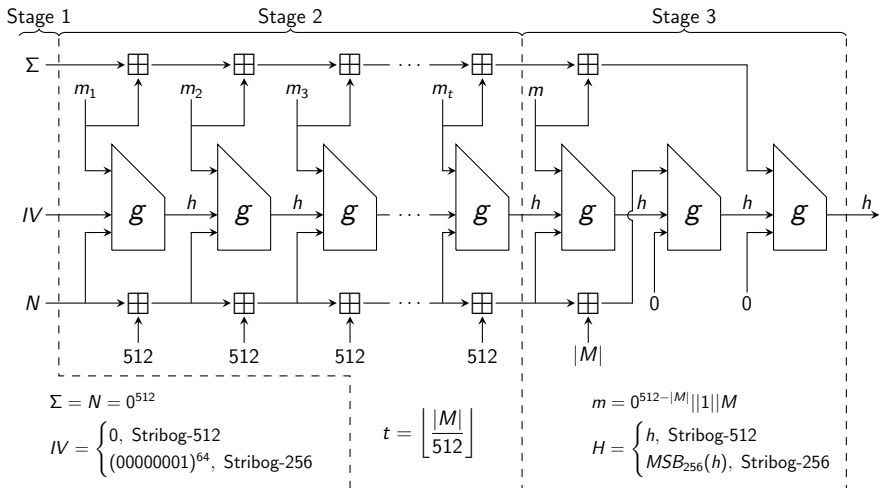
Algebraic Aspects of the Russian Hash Standard GOST R 34.11-2012

Oleksandr Kazymyrov Valentyna Kazymyrova

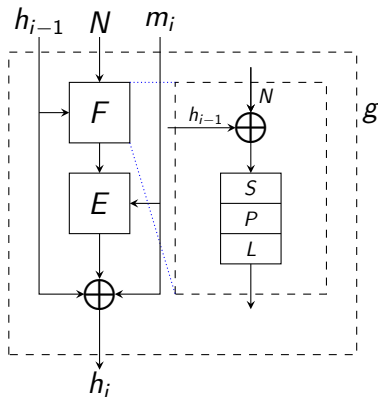
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June 25, 2013

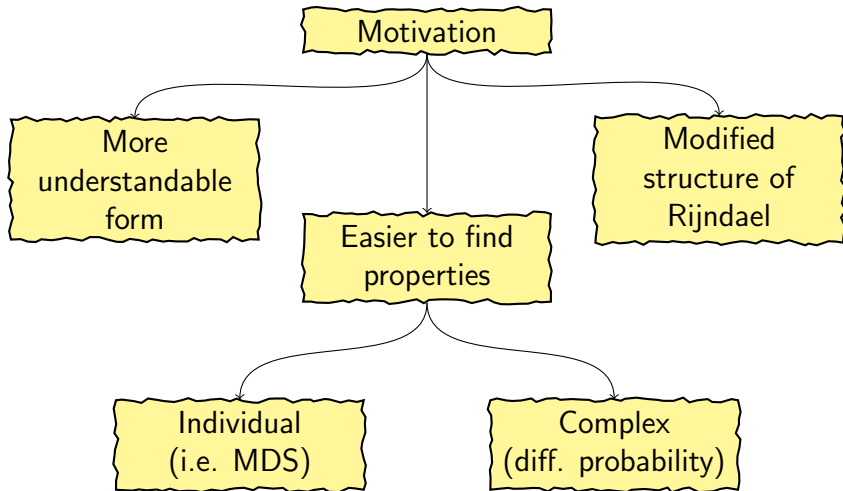
Hash function Stribog



Construction of the compression function g



Motivation



State representation

An alternative representation

- Reverse input bits
- AES-like transformations (the state as in Grøstl)
- Reverse output bits



Transposition and SubBytes operations

- **Transposition** is invariant operation.
- Substitution has the form $F(x) = D \circ G \circ D(x)$ for linearized polynomial $D : \mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^n}$.

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	3F	FB	D7	E0	9F	E5	A8	04	97	07	AD	87	A0	B5	4C	9A
1	DF	EB	4F	0C	81	58	CF	D3	E8	3B	FD	B1	60	31	B6	8B
2	F3	7C	57	61	47	78	08	B4	C9	5E	10	32	C7	E4	FF	67
3	C4	3E	BF	11	D1	26	B9	7D	28	72	39	53	FE	96	C3	9C
4	BB	24	34	CD	A6	06	69	E6	0F	37	70	C1	40	62	98	2E
5	5F	6B	16	D6	3C	1C	1E	A4	8F	14	C8	55	B7	A5	63	F5
6	8C	C2	12	B8	F7	46	59	90	99	0D	6E	1F	F1	AA	51	2D
7	20	9D	73	E7	71	64	4D	36	FA	50	BA	A1	CB	A9	B0	C6
8	77	AF	2C	1A	18	E9	85	8E	EE	F0	0E	D8	21	A2	AE	65
9	23	9E	54	EC	38	1D	89	D9	6C	17	4E	CA	D0	C5	2A	66
A	76	15	13	35	3A	00	DE	D4	74	29	30	FC	56	7A	AC	2F
B	A3	44	5C	9B	80	F9	79	A7	B3	CC	ED	1B	2B	AB	BD	D2
C	88	95	8A	02	5A	CE	94	25	DB	7B	6A	92	75	49	BC	4B
D	5B	6F	45	27	42	41	F6	0B	DD	0A	E2	09	19	BE	01	43
E	68	93	D5	EF	84	22	E3	DA	5D	3D	48	7F	05	F4	7E	03
F	B2	C0	33	91	F2	82	8D	4A	83	52	E1	86	F8	DC	EA	6D

Table : The Substitution F for AES-like Description

Representation of MixColumns

Let $L : \mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^n}$ be a linear function of the form

$$L(x) = \sum_{i=0}^{n-1} \delta_i x^{2^i}, \quad \delta_i \in \mathbb{F}_{2^n}.$$

Proposition (Paper VII)

Any linear function $L : \mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^m}$ can be converted to a matrix with the complexity $O(n)$.

$$L(x) = \delta x, \quad \delta_i = 0, \text{ for } 1 \leq i \leq n - 1.$$

Representation of MixColumns

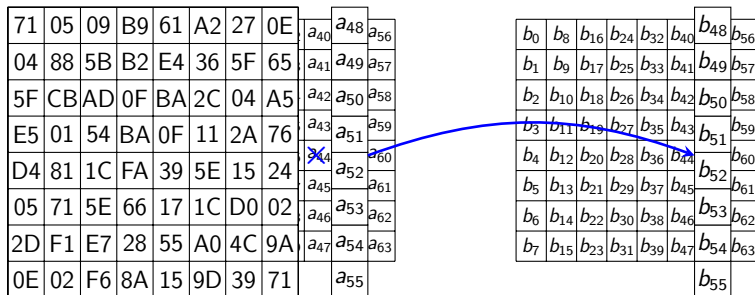
The main steps of proposed algorithm for obtaining MDS matrix over \mathbb{F}_{2^8} from 64×64 matrix over \mathbb{F}_2

- for every irreducible polynomial (30)
 - convert each of 8×8 submatrices to the element of the field
 - check MDS property of the resulting matrix

Additional transformation

It is necessary to **transpose matrix of Stribog** before applying the algorithm.

MixColumns



Multiplying the vector by the constant 8×8 matrix G over \mathbb{F}_{2^8} with the primitive polynomial $f(x) = x^8 + x^6 + x^5 + x^4 + 1$

$$B = G \cdot A$$

Summary

- GOST R 34.11-2012 is based on GOST 34.11-94 as well as on Whirlpool/Grøstl/AES.
- Proposed method to reconstruct initial representation has many application fields.
- Nonlinear dependence of the performance and the message length.
 - More details on <https://github.com/okazymyrov>

Extended Criterion for Absence of Fixed Points

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CTCrypt'13
June 25, 2013

Properties of substitutions

Definition

Substitution boxes (*S*-boxes) map an n -bit input message to an m -bit output message.

- Minimum of Algebraic Degree
- Balancedness
- Nonlinearity
- Correlation Immunity
- δ -uniformity
- Cycle Structure
- Algebraic Immunity
- Absolute Indicator
- Absence of Fixed Points
- Propagation Criterion
- Sum-of-squares indicator
- ...

Definitions and notations

Definition

A substitution must not have fixed point, i.e.

$$F(a) \neq a, \quad \forall a \in \mathbb{F}_2^n.$$

Definition

Two ciphers E_i and E_j are isomorphic to each other if there exist invertible maps $\phi : x^i \mapsto x^j$, $\psi : y^i \mapsto y^j$ and $\chi : k^i \mapsto k^j$ such that $y^i = E_i(x^i, k^i)$ and $y^j = E_j(x^j, k^j)$ are equal for all x^i, k^i, x^j and k^j .

Basic functions of AES

The round function consists of four functions

- AddRoundKey (σ_k)
- SubBytes (γ)
- ShiftRows (π)
- MixColumns (θ)

$$E_K(M) = \sigma_{k_{r+1}} \circ \pi \circ \gamma \circ \prod_{i=2}^r (\sigma_{k_i} \circ \theta \circ \pi \circ \gamma) \circ \sigma_{k_1}(M).$$

Both **MixColumns** and **ShiftRows** are linear transformations with respect to XOR

$$\begin{aligned}\theta(x + y) &= \theta(y) + \theta(y); \\ \pi(x + y) &= \pi(y) + \pi(y).\end{aligned}$$

Isomorphic algorithm to AES

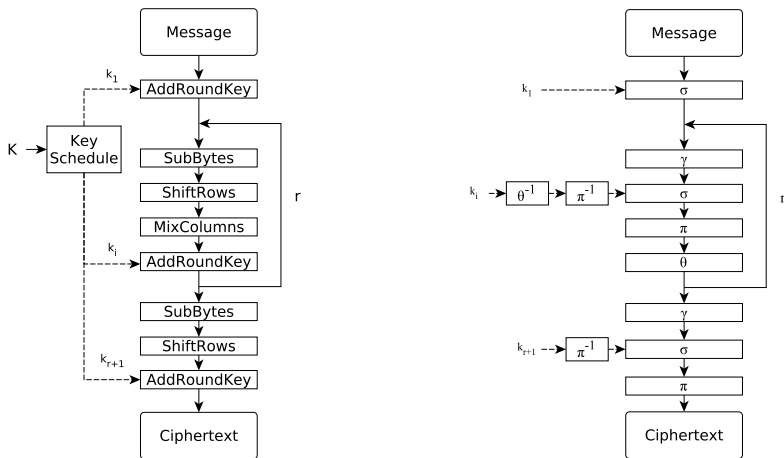


Figure : Encryption Algorithm

Isomorphic algorithm to AES

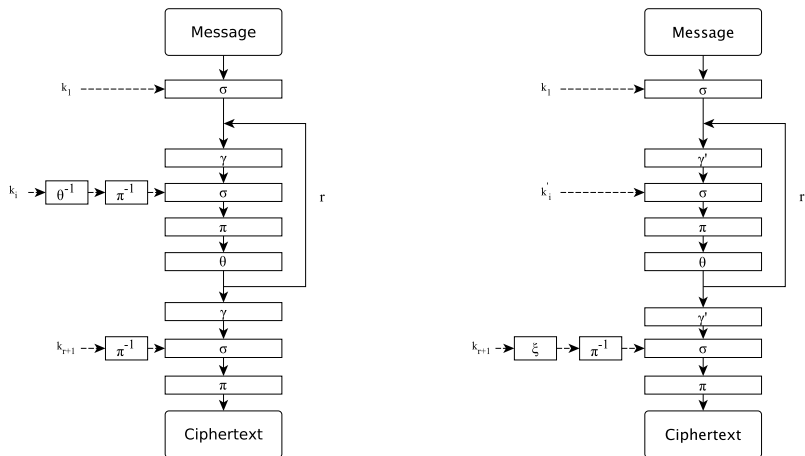


Figure : Encryption Algorithm

Summary

Isomorphic ciphers allow to

- Show redundancy of the last ShiftRow operation of the AES.
- Prove/disprove necessity of some characteristics of substitutions.
- Introduce new criterion for several substitutions.
- Show advantages of addition modulo 2^n in comparison with XOR operation.

Proposition

At least absence of fixed points criterion should be reviewed with other components of ciphers.

State space cryptanalysis of the MICKEY cipher

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ITA'13
February 11, 2013

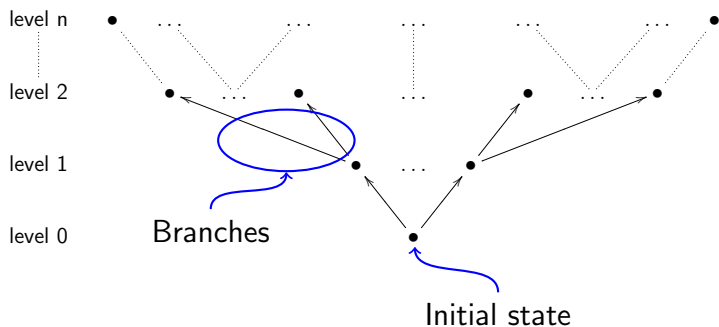
A general attack scenario on stream ciphers

- Recover states of registers (Berlekamp-Massey algorithm, algebraic attack, Rønjom-Helleseth attack)
- Find the key based on the known state
 - **allows to estimate the number of possible states**

Note

In some stream ciphers the first step is sufficient to find the key

Tree of backward states



Degree probabilities

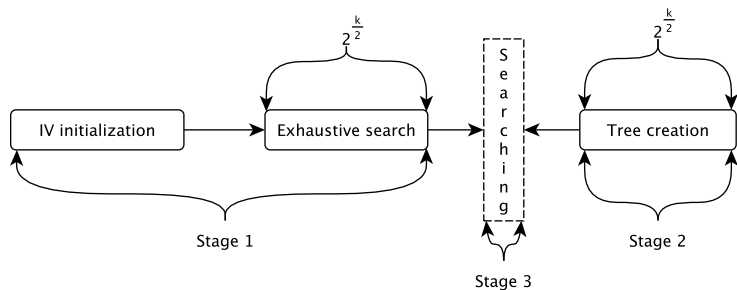
Degree	Key/IV load		Preclock mode		KG	
	80 v2	128 v2	80 v2	128 v2	80 v2	128 v2
0	0.2773	0.2186	0.3052	0.29	0.3041	0.3038
1	0.00001	0.1047	0.4345	0.4534	0.4323	0.4154
2	0.4331	0.3753	0.2523	0.2256	0.2558	0.2698
3	0.00002	0.1029	-	0.0289	-	-
4	0.28	0.1783	0.008	0.0021	0.0079	0.0111
6	0.00007	0.0203	-	-	-	-
8	0.0095	-	-	-	-	-

Determination of key bits based on a backward states tree

Level	Bit probability			
	MICKEY-80 v2		MICKEY-128 v2	
	1	0	1	0
1	0.5	0.5	1	0
2	0.5	0.5	0.5	0.5
3	0.5	0.5	0	1
4	0.5	0.5	0.5	0.5
5	0.4857	0.5143	0.5	0.5

$$O(2^{126} + 2^t) \stackrel{t \ll 126}{\approx} O(2^{126}) < O(2^{128})$$

Meet-in-the-middle attack on MICKEY



$$O(2^{\frac{k}{2}+2}) = O_d(2^{\frac{k}{2}}) + O_i(2^{\frac{k}{2}}) + O_f(2^{\frac{k}{2}})$$

Identical key-streams for different key/IV pairs

Let z_i^h be i -th bit of a key-stream for h -th pair of (K_h, IV_h) . Suppose also that

$$K_1 = k_0, k_1, \dots, k_{n-1}$$

Then it is possible to find such (K_1, IV_1) and (K_2, IV_2) for which the states of registers will differ by one clock and the key-streams have the property

$$z_i^2 = z_{i+1}^1$$

An example of key/IV with shifted key-streams

$$K_1 = \{d3, ec, f0, 84, 8a, 1d, b1, b7, 4a, dd\}$$

$$IV_1 = \{58, e5, 77, 0a, 9c, a2, 34, c7, cd, 5e\} \text{ (79bits)}$$

$$K_2 = \{a7, d9, e1, 09, 14, 3b, 63, 6e, 95, ba\}$$

$$IV_2 = \{58, e5, 77, 0a, 9c, a2, 34, c7, cd, 5f\} \text{ (80bits)}$$

$$Z_1 = \{0, B7, 61, 27, 92, C5, 85, 91, 51, 18, 2A, D6, 7C, 8C, C8, C7, 04\}$$

$$Z_2 = \{ B7, 61, 27, 92, C5, 85, 91, 51, 18, 2A, D6, 7C, 8C, C8, C7, 04, 1\}$$

Summary

- Proposed method allows to estimate degrees' probability at the design stage of MICKEY-like ciphers.
- Stepping backwards in the state space of MICKEY is possible and feasible in all modes including key/IV load mode.
- A minor change in the feedback function of the R-register involve dramatically changes in cycles.
- Thus, it is possible to justify the choice of the encryption algorithm parameters.
- Several practical attack scenarios based on known states were proposed.

Verification of EA-equivalence for Vectorial Boolean Functions

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WAIFI'12
July 17, 2012

Open problems

1. Verification of EA-equivalence for arbitrary functions.
2. For given functions F and G , find affine permutations A_1, A_2 and a linear function L_3 such that

$$F(x) = A_1 \circ G \circ A_2(x) + L_3(x)$$

Complexity of exhaustive search for $F, G : \mathbb{F}_2^n \mapsto \mathbb{F}_2^n$ equals $O\left(2^{3n^2+2n}\right)$. For $n = 6$ the complexity is already 2^{120} .

Summary

Restricted EA-equivalence	Complexity	$G(x)$
$F(x) = M_1 \cdot G(M_2 \cdot x)$	$O(n^2 \cdot 2^n)$	P
$F(x) = M_1 \cdot G(M_2 \cdot x \oplus V_2) \oplus V_1$	$O(n \cdot 2^{2n})$	P
$F(x) = M_1 \cdot G(x \oplus V_2) \oplus V_1$	$O(2^{2n+1})$	†
$F(x) = M_1 \cdot G(x \oplus V_2) \oplus V_1$	$O(n \cdot 2^{3n})$	A
$F(x) = G(M_2 \cdot x \oplus V_2) \oplus V_1$	$O(n \cdot 2^n)$	P
$F(x) = G(x \oplus V_2) \oplus M_3 \cdot x \oplus V_1$	$O(n \cdot 2^n)$	A
$F(x) = M_1 \cdot G(x \oplus V_2) \oplus M_3 \cdot x \oplus V_1$	$O(2^{2n+1})$	‡
$F(x) = M_1 \cdot G(x \oplus V_2) \oplus M_3 \cdot x \oplus V_1$	$O(n \cdot 2^{3n})$	A

† - G is under condition $\{2^i \mid 0 \leq i \leq m-1\} \subset \text{img}(G')$ where

$$G'(x) = G(x) + G(0).$$

‡ - G is under condition $\{2^i \mid 0 \leq i \leq m-1\} \subset \text{img}(G')$ where

$$G'(x) = G(x) \oplus L_G(x) \oplus G(0).$$

Conclusions

- Cryptanalytic methods applied to MICKEY, DES and MiniAES can be used to improve cryptographic properties of prospective ciphers
- In the post-AES era many cryptoprimitives providing high-level security use random substitutions
- The new heuristic method to generate S-boxes was proposed
 - Surpass analogues used in Russian and Belorussian standards

Conclusions

- Several methods to check the REA-equivalence of two binary nonlinear mappings have been proposed
- Isomorphic representations open new directions in cryptanalysis
 - Nonlinear mappings
 - Overall design principles
- The main practical result is the designed software for effective generation and calculation of indicators of arbitrary nonlinear binary mappings.

Methods and Tools for Analysis of Symmetric Cryptographic Primitives

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14th of October, 2014